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Sound wave structures downstream of pseudo-steady weak and strong Mach reflections

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Sound wave structures, downstream of moving incident shocks reflecting from straight compressive wedges, are analysed for both weak and strong Mach reflections (MR) using existing experiments. It is shown that the reflected waves can be well described by using the acoustic criterion or the weak oblique shock approximation, when the classical three-shock theory gives forward-facing reflected shock solutions. The predicted triple-point trajectory angles are found to be in close agreement with the experiments. The distinction between the applicabilities of the two methods is given by an analytically defined 'smallness' for the angle of reflecting wedges. The physics of the success of the two methods is discussed. It is concluded that forward-facing reflected shock solutions of pseudo-steady MR should be ruled out physically because sound waves cannot coalesce into Mach waves that propagate upstream of the triple point. In their place, MR-like phenomena occur with the reflected waves being normal Mach waves or finite compression waves for 'small' or 'not-small' reflecting wedge angles, respectively, and they are classified as the first- or second-kind von Neumann reflections, respectively. Boundaries separating regimes between the first and second kinds of von Neumann reflections, and backward-facing MR are determined.

1. Introduction

The problem of pseudo-steady oblique shock reflection has been investigated analytically and experimentally by many researchers since Mach (1878) presented his work on the subject more than a century ago. Von Neumann (1945) formulated the classical two- and three-shock theories for regular reflection (RR) and Mach reflection (MR) in pseudo-steady frames of reference relative to the reflection point and triple point, respectively, the clarity of which still pervades thinking about the problem today. He also found that the strength of the incident oblique shock becomes a necessary factor for determining the transition between RR and MR. Using a property of the shock polar diagram, he gave a rigorous definition of the boundary between weak and strong incident shocks. The limiting condition for a perfect gas with the ratio of specific heats γ of 1.4 is equivalent to an incident propagating shock Mach number of 1.46.

However, it has been known since the important shock reflection experiment of Smith (1945) in pseudo-steady flows in air and the discussion of them by Bleakney & Taub (1949) that this classical three-shock theory almost always failed to agree with experiment for weak MR. Perhaps the most violent disagreement, as noted by them, is that Mach reflections exist where there are no non-trivial solutions from the three-shock theory when the incident shock becomes sufficiently weak. This was called a

MR-like phenomenon in a quite recent international workshop (Sakurai *et al.* 1989) on the subject of weak MR. The theory begins to provide physically acceptable solutions and gives some less-than-modest agreement with experiment when the incident shock Mach number M_s approaches the weak/strong boundary and the reflecting wedge angle nears the transition from MR to RR. Remarkably, the same theory succeeds brilliantly for many strong MR (without real-gas effects) when M_s moves away from the weak/strong boundary. For RR, the von Neumann two-shock theory in general succeeds for both weak and strong incident shocks, except near the transition to MR where RR apparently persists beyond the detachment boundary line in pseudo-steady flows. The almost complete failure of the theory for weak MR, its success for many strong MR, and the apparent persistence of RR into the regime of MR in pseudo-steady flows have been termed 'the von Neumann paradox' in the literature (e.g. Griffith 1981; Colella & Henderson 1990; and Ben-Dor 1992). In seeking an explanation for the incapability of the three-shock theory to adequately describe weak MR, fundamental assumptions of the theory, such as viscosity, finite thickness in reflected/Mach shocks, non-self-similarity, and non-uniformity, must be reviewed as have been reported, mostly to not much avail, by Sakurai (1964), Sternberg (1959), Henderson & Siegenthaler (1980) and Sakurai *et al.* (1989). In fact, these effects are always there for both weak and strong MR. If they are of importance for adequately describing weak MR, why does the same theory brilliantly describe most of the strong MR (where the reflected waves are also often curved) without considering them? It is worth mentioning reported evidence of viscous and unsteady effects found in low-density experiments (initial pressures of a few torrs) where the triple point does not necessarily travel along a linear trajectory through the leading edge of a reflecting wedge (Walenta 1983; Schmidt 1985). These effects, however, are considered negligible for the high-density experiments to be discussed in the present work.

Recently, Colella & Henderson (1990) numerically simulated weak MR and MR-like phenomena, using adaptive mesh refinement techniques, by solving the unsteady two-dimensional Euler equation. They showed that there is one kind of weak irregular wave reflection called a von Neumann reflection (vNR). The reflected wave of vNR is not a shock but a smoothly distributed compression wave of finite thickness, and the incident and Mach shocks appear to form a single wave with a continuously turning tangent. Their analysis therefore explains why the conventional theory cannot be applied to vNR. They further hypothesized that a vNR exists under two conditions: the first, by experiment, is when the predicted angle between the reflected shock and slipstream from the three-shock theory becomes greater than 90°; the second is when the three-shock theory has no physically acceptable solutions (Henderson 1987). On the other hand, Olim & Dewey (1992) confirmed the observations of other workers that the three-shock theory is unable to describe MR of incident shocks with strengths between Mach number 1.1 and 1.5, and for wedge angles not far from those at which transition from RR to MR occurs. The resulting three-shock configuration is such that the reflected wave is supersonic and has the characteristics of a shock wave, and that there is a measureable discontinuity in the slopes of the incident and Mach stem shocks at the triple point. In this work, we refer to the existing unexplained large discrepancy between the classical three-shock theory and experiment for pseudo-steady MR as a consequence of the occurrence of MR-like phenomena.

The questions now raised are the following: Does the hypothesized vNR actually correspond to observed MR-like phenomena? Does a general vNR exist without the restriction of a smooth turning tangent of the incident and Mach shocks at the triple point? Can the solution of a vNR be predicted from a modified three-shock theory instead of from a sophisticated numerical calculation? The object of this paper is to seek explanations for the almost complete and partial failures of the classical three-shock theory in describing MR-like phenomena. We still employ essentially the same von Neumann theory with the exception of allowing degenerate reflected shock solutions. This gasdynamics discontinuity approach is considered more appropriate than Whitham's ray shock method (Whitham 1957), because Whitham did not consider the existence of the reflected shock and slipstream which we believe is important for the case of MR. Henderson (1980) made a quite detailed comparison between the theory and his pseudo-steady MR data. He found that Whitham's theory does not, in general, explain the observed MR well and it is not applicable to RR. The prime consideration in this work is to clarify what are physically acceptable and unacceptable reflected shock solutions of pseudo-steady MR. Since it is known that the theory explains most of the strong MR well, one should look for degenerate reflected shock solutions when incident shock waves become weak. To these ends, we analyse sound wave structures downstream of existing, observed pseudo-steady MR in addition to calculating possible MR solutions. By analogy with simple sound generation by a supersonically flying wedge, it is shown that the source of reflected disturbances leading to the formation of a reflected shock wave in pseudo-steady MR originates from the collisions between fluid particles behind the incident shock and the slipstream, as the triple point propagates upstream. It is noted that uniform sound speed immediately behind the incident shock is used to construct sound waves characterizing the initial stage of the converging process of the downstream Mach waves, which may lead to the formation of a reflected shock wave. This initial development of downstream sound structures helps clarify physically acceptable and unacceptable reflected shock solutions of pseudo-steady MR.

It is important to remark on Hornung's information condition concept (Hornung 1986), by which he argues, for the transition between RR and MR, that MR (which displays a characteristic length at the reflection point) is not possible when the information about the characteristic length of the problem does not reach the reflection point unless the flow behind the reflected shock becomes subsonic relative to the reflection point. However, in line with the analysis of sound structures proposed here, one may further argue that RR is not possible when the information about the wedge corner, i.e. sound waves generated there, reaches the reflection point unless the flow behind the incident shock is supersonic relative to the reflection point. This simple sound consideration addresses the issue of the persistence of RR into the regime of MR from the viewpoint of a sufficient condition for the existence of MR. Finally, it is worthwhile to remark on the various linearized solutions for the shock wave pattern of pseudo-steady MR at nearly glancing incidences obtained by Bargmann (1945), Lighthill (1949), and Ting & Ludloff (1951) by solving the Euler equation, as discussed by Fletcher, Taub & Bleakney (1951). Their discussion was motivated by attempting to clarify the large discrepancies between observed MR-like phenomena and predictions from the three-shock theory. Essentially, these authors assumed a zero-order solution and determined a first-order solution in terms of it. The zero-order solution used is a trivial one in which the reflected wave is assumed to be a sound signal and hence of zero strength. They then deduced that on the reflected shock the pressure variation across it is zero. Thus, in the neighbourhood of the triple point, these first-order solutions correspond to the assumed trivial solution. In addition to this limitation restricted by the asymptotic solution of the linearized

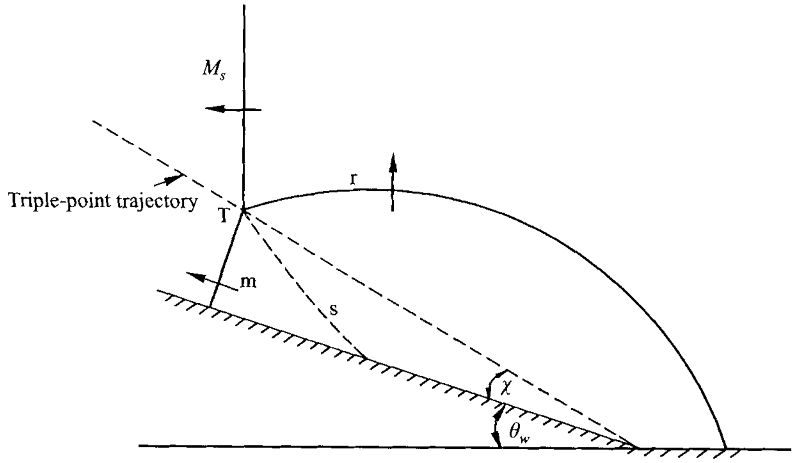

FIGURE 1. Schematic illustration of a propagating MR over an oblique wall. i, incident shock; r, reflected shock; m, Mach stem; s, slipstream; T, triple point; θ_w, wedge angle; χ, triple-point trajectory angle; M_s, incident shock Mach number.

differential equations, there are two major difficulties with this approach. The first is that the locations and strengths of the reflected and Mach shocks which are to be determined by the analysis also specify the boundary conditions for the problem. The second is the inadequacy of the method for describing the slipstream which, in the case of weak MR, seems experimentally to be a sharp density discontinuity, as was reported by them.

This paper is organized in the following way. First, the classical three-shock theory is briefly reviewed. Emphasis is given to the calculation of forward-facing reflected shock solutions of pseudo-steady MR. The von Neumann paradox of weak MR is discussed in §3. Attention is paid to the coincidence of the worsening of the three-shock theory in describing pseudo-steady MR with the requirement of forward-facing reflected shock solutions from the theory. Analyses of sound wave structures downstream of existing, observed pseudo-steady MR and MR-like phenomena are given in §4 for five representative cases. Attention is centred on the process of generation of reflected sound disturbances giving rise to coalescing Mach waves downstream of incident oblique shock waves. Comparisons between computed and measured triple-point trajectory angles and reflected shock wave angles along with the comparisons of observed reflected wave angles with constructed downstream sound structures are made to assess physically acceptable and unacceptable multiple reflection solutions of pseudo-steady MR. In the process, the success of the acoustic criterion and the weak oblique shock approximation for describing reflected waves of MR-like phenomena are verified. We shall present pressure–deflection shock and compression polar solutions of pseudo-steady MR as supporting data in addition to the experiments.

In order to keep the physics as simple as possible, only existing experiments of single Mach reflection type over straight wedges with insignificant real-gas effects are selected for analysis in the present study. The initial pressure in the chosen shock-tube experiments is relatively high. This has the effect of enhancing the quality of flow visualization pictures and minimizing the relatively unimportant viscous displacement effect.

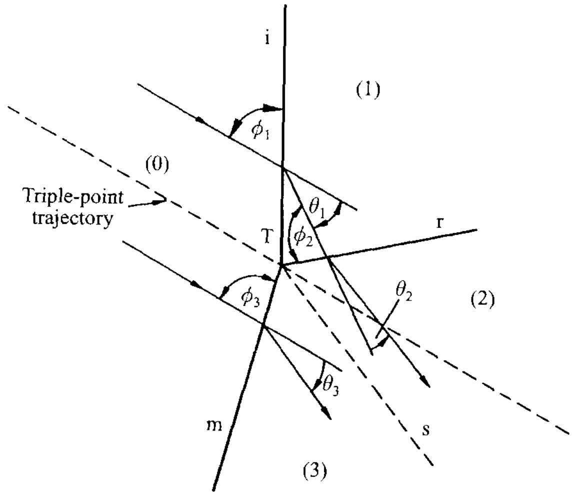

FIGURE 2. The wave configuration of a MR viewed from an observer moving with the triple point T: (0)–(3), flow states; ϕ, angle of incidence; θ, deflection angle; i, r, m, and s are defined in the caption of figure 1.

2. The von Neumann three-shock theory

A typical pseudo-steady single Mach reflection, as obtained in shock tube experiments over straight wedges, is shown schematically in figure 1: there are four discontinuities – the incident shock i, the reflected shock r, the Mach stem m, and the slipstream s – which coincide at the triple-point T, making a triple point trajectory angle χ with the wedge surface having an inclination angle θ_w. Since the analytical formulation of the von Neumann three-shock theory for a perfect gas is well-known (e.g. Ben-Dor 1992), the theory is not discussed in detail here. Basically, it consists of 12 gasdynamics discontinuity conservation equations describing incident, reflected, and Mach stem shocks; and two pressure and flow deflection identity equations describing a contact discontinuity separating the reflected and Mach shocks that are solved for 14 flow variables behind these three confluent shock waves. The regions bounded by the thin and planar discontinuities are assumed uniform and in thermodynamic equilibrium. Fourteen typical flow variables are p_1 (static pressure), T_1 (static temperature), u_1 (flow velocity), ϕ_2 (shock wave angle), θ_1 (deflection angle), p_2, T_2, u_2, θ_2, p_3, T_3, u_3, ϕ_3, θ_3. Subscripts 0, 1, 2, and 3 refer to the flow states as defined in figure 2. These subscripts so defined are used throughout this paper.

The solution for this system of nonlinear algebraic equations of pseudo-steady MR must be sought by specifying upstream conditions p_0, T_0, u_0, and ϕ_1 with an assumed initial χ. Since this angle does not enter into any of the 12 conservation equations explicitly, the solution procedure is an iterative one by determining χ until the boundary condition describing the Mach stem and wedge surface is satisfied. The usual boundary condition is that the Mach stem is straight and perpendicular to the wall, so it pulls fluid particles parallel to the wall behind it (Law & Glass 1971). This boundary condition is experimentally verified in the five MR cases to be discussed in §4, and it is referred to as the normal, straight Mach stem boundary condition throughout this paper.

For a given wedge angle and γ, the predicted χ and reflected shock angle ϕ_2 increase as the incident shock Mach number M_s decreases. This is shown in figures 3 and 4

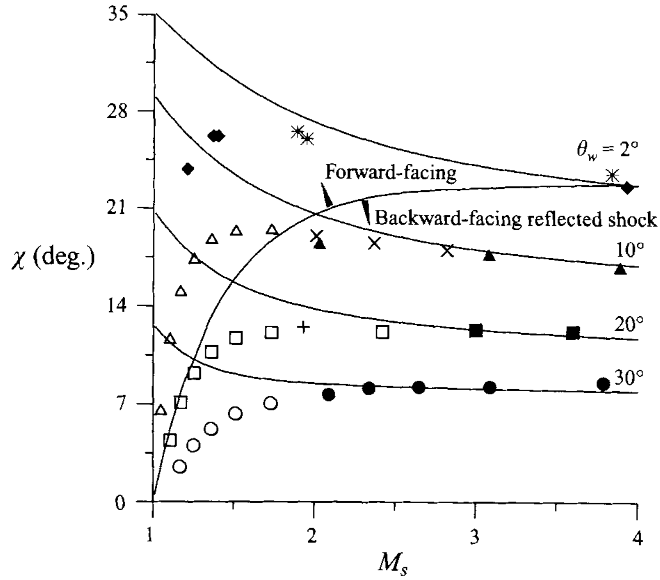

FIGURE 3. Variations of the three-shock theoretical predictions and experimental observations of χ with M_s for four different wedge angles. Solid symbols: Deschambault (1984, air); open symbols: Smith (1945, air); cross symbols: Ben-Dor (1978, nitrogen). Circles: 30°; squares (or +): 20°; triangles (or ×): 10°; diamonds (or *): 2°.

for four different reflecting wedge angles in air for $1.0 < M_s \leqslant 4.0$. A reflected shock is classified to be backward- (or forward-) facing when the reflected shock angle, i.e. the angle between the incident flow direction and the emanating direction of the reflected shock, is less (or greater) than 90°. For a limiting weak/strong M_s of 1.46 with $\gamma = 1.4$, a backward-facing reflected shock solution can be obtained only when the wedge angle is larger than 21.1°. Likewise, there exists, for given θ_w and γ, a changeover M_s below which the reflected shock of MR becomes forward-facing. This M_s is called the critical incident shock Mach number in the present work. For $\gamma = 1.4$, the critical M_s is 2.01 for a 10° wedge. It becomes 4.01 or 1.49 for 2° or 20° wedges, respectively. A boundary line separating backward- from forward-facing reflected shock solutions is shown in figure 3. One can see that the critical M_s increases rapidly as θ_w decreases from 10° to 2°.

It is noteworthy that a forward-facing reflected shock solution of pseudo-steady MR always exists when the incident shock becomes sufficiently weak, even though it has been reported otherwise in the literature (e.g. Dewey *et al.* 1989; Ben-Dor 1992) that a MR solution is theoretically unobtainable because the reflected shock polar does not intersect the incident polar. Henderson (1987) defines an incident shock as 'extremely weak' when $M_s \leqslant 1.083$ ($\gamma = 1.4$), and he shows that only RR and a hypothetical 'continuous wave reflection' can exist for the extremely weak shocks. The continuous wave reflection is assumed when the three-shock theory cannot provide a MR solution. He then reported that the transition from RR to the continuous wave reflection occurs when ϕ_1 increases to 54.29° for $M_s = 1.083$. However, it can be seen from the shock polar solution given in figure 5 that a forward-facing reflected shock solution from the three-shock theory with a predicted χ (27.88°), satisfying the normal, straight Mach stem boundary condition, is indeed obtained at $\phi_1 = 52.12°$ for the same incident shock reflecting from a 10° wedge.

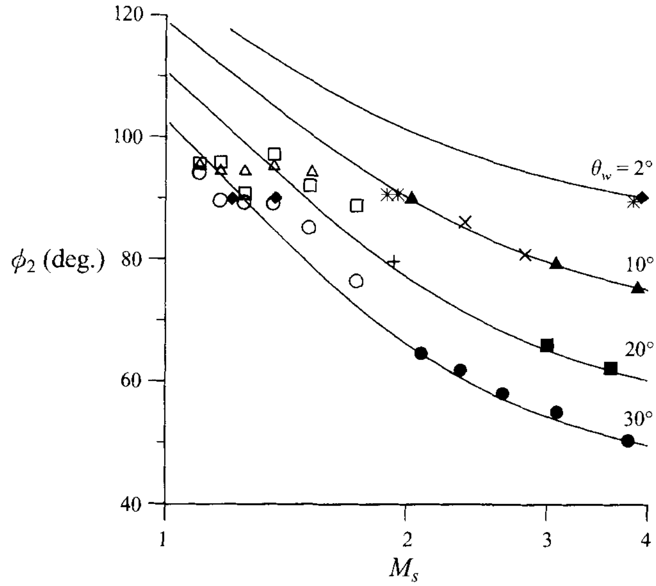

FIGURE 4. Variations of the three-shock theoretical predictions and experimental observations of ϕ_2 with the incident shock Mach number for four different wedge angles. For symbols see the caption to figure 3.

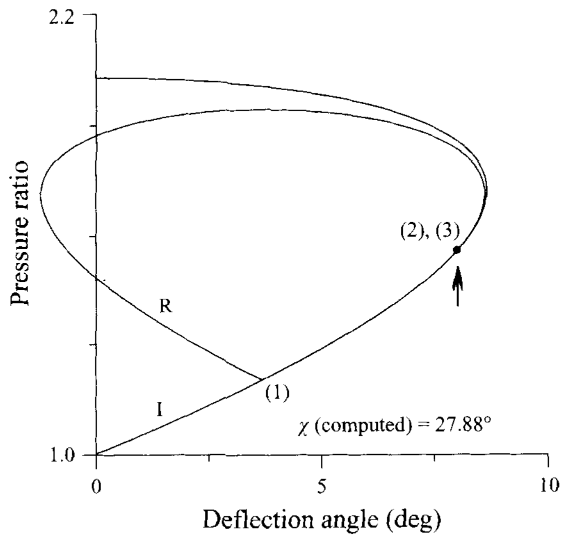

FIGURE 5. The shock polar representation of a forward-facing reflected shock solution of a weak MR ($M_s = 1.083, \theta_w = 10°$). The flow states 1–3 are defined in figures 1 and 2.

3. The von Neumann paradox of weak Mach reflections

Despite the fact that both backward- and forward-facing reflected shock waves are theoretically permissible solutions of pseudo-steady MR, the observed reflected waves are never inclined forward of the triple point. Actually, even a normal shock reflected off the triple point does not materialize in reality. In its place, a near-forward-facing MR-like phenomenon with a visible slipstream is always observed experimentally (e.g. Bleakney & Taub 1949; Colella & Henderson 1990; and Ben-Dor 1992). As the critical M_s has values obviously not limited to weak shock waves, it is not surprising to find that MR-like phenomena exist for strong incident shocks reflecting over relatively small wedge angles.

Figures 3 and 4 show comparisons between the predictions from the classical three-shock theory and experiment for χ and ϕ_2 for four reflecting wedge angles. M_s is limited to not exceed 4 to avoid any significant real-gas effects in air and nitrogen. The experimental data were taken from the works of Deschambault 1984 (solid symbols), Ben-Dor 1978 (cross symbols), and Smith 1945 (open symbols). It is found that some of the χ and ϕ_2 reported by Deschambault were not accurately measured. Reflected shock angles should be measured close to the triple point. These angles were remeasured from his 16.5 cm by 12.3 cm size interferometric pictures with a vernier scale from a Mutoh protractor. The accuracy of the scale is within one twelfth of one degree. The absolute error in determining M_s in his experiments was reported (Deschambault 1984) as 0.01 for $M_s = 1.1$ and 0.22 for $M_s = 10.0$. The uncertainty in the measurement of χ and ϕ_2 from his interferograms is estimated at $\pm 0.15°$ and $\pm 0.5°$, respectively. This angle measurement method is also used for determining various wave angles in observed MR configurations to be discussed in §4. The relative error of M_s in Ben-Dor's experiments was estimated (Ben-Dor 1978) to be within 1.25% at $M_s = 2$. We could not find an estimate for the accuracy of M_s in Smith's experiments, and there are no photographs for the experimental data reported in his work. Smith's data shown in figures 3 and 4 were therefore taken directly from his report.

It can be seen from these two figures that, except for the 2° wedge where data are not available for $2 < M_s < 4$, the agreement between the experiment and three-shock theory for both angles is excellent when M_s is greater than about 2.3. The agreement begins to deviate when it becomes close to 2. It then worsens rapidly as M_s further decreases towards its critical value where ϕ_2 approaches 90° (i.e. becomes forward-facing). In fact, the trend of the variation between the predicted and measured χ moves in increasingly opposite directions as M_s further decreases below its critical value! The fact that a triple-point trajectory separates two differing flows behind incident and Mach shocks while a reflected shock may degenerate to a Mach wave suggests that χ is more appropriate than ϕ_2 for characterizing a MR. Therefore, it is hard to escape the notion of a paradox when considering why the three-shock theory fails so miserably for weak MR while the same theory succeeds brilliantly for many strong MR. One also learns from these comparisons that the failure of the theory is probably associated with the requirement of forward-facing reflected shock solutions as M_s further decreases for a given wedge angle.

A comparison between figures 3 and 4 shows that there is an intrinsic difference between the trends in the variation of the measured χ and ϕ_2 for weak MR, with the former continuing to decrease while the latter levels off, as M_s decreases below its critical value. One further notices that the measured ϕ_2 does not even seem to depend on the wedge angle when the predicted values approach or exceed 90°. In fact, from the flow visualization pictures of MR available in current research, it is found that observed MR-like phenomena always exhibit near-limiting 90° reflected wave configurations in the vicinity of the triple point. These observations suggest that viscosity or flow non-uniformity may not be attributed to the causes of the von Neumann paradox of weak MR. In the following, this problem is examined from the viewpoint of analysing sound wave structures downstream of incident propagating oblique shock waves in addition to calculating multiply possible MR solutions. It is shown that the sound structures provide physical bases for using the acoustic criterion or the weak oblique shock approximation for accurately describing observed reflected waves in MR-like phenomena. They also lead to an important conclusion for the physical non-existence of forward-facing reflected shock waves of pseudo-steady MR.

4. Analysis

Oblique shock reflections are nonlinear phenomena where multiple solutions often exist for a given boundary condition. Therefore, it is logical to first look at the dynamic process that may lead to the formation of a shock wave. Figure 6 shows a wedge propagating in a compressible medium. When the relative velocity of fluid particles and the wedge exceeds the local fluid sound speed, sound waves generated by the collisions between fluid particles and the wedge surface give rise to Mach waves. This flying wedge can be regarded as a heterogeneous source of sound generation in an otherwise homogeneous flow field. Subsequent addition of individual Mach waves emitted from the wedge surface forms compression waves that may then interact nonlinearly with each other resulting in the formation of an induced shock wave. The dynamics of this shock-inducing flow field is then determined by the self-similar sound-generating and sound-propagating process, which is characterized by the two competing velocities: the relative velocity of the upstream fluid and the propagating wedge, and the local fluid sound speed. The former velocity essentially constitutes a path of sound generation centres, along which the sound structure of a compressible flow field can be drawn. A sound generation centre is the location from which sound disturbances propagate at a given time. It is believed that this sound-originated from-Mach-to-shock formation process can be applied directly to the analysis of the sound structure downstream of a pseudo-steady MR.

The origin of sound generation responsible for the formation of a reflected shock wave in pseudo-steady MR can be regarded as the collisions between fluid particles behind the incident shock and the induced slipstream as the triple point propagates upstream. The self-similarly growing slipstream stemming from the triple point plays an effective role as the Mach-wave-emitting surface of the flying wedge for fluid particles downstream of the incident shock wave. The key consideration in the construction of sound waves downstream of a pseudo-steady MR is to recognize that, before a reflected shock is formed, radially propagating sound waves so generated are convected uniformly by the actual fluid particle velocity behind the incident shock. Thus, the path of a propagating triple point should be available, in addition to a given combination of M_s and θ_w, before a downstream sound structure is analysed. It is important that uniform sound speed immediately behind the incident shock is used to construct the downstream sound waves, irrespective of the existence of a reflected shock wave. This sound construction can be regarded as carrying out a thought experiment for simulating an initial sound development in the coalescing process of the downstream Mach waves, when an incident shock first encounters a wedge corner. It is known that the shock reflection process over plane wedges in shock tubes has been found by many experimentists to be self-similar. Therefore, the downstream sound waves can be constructed at locations not near the wedge corner in order to compare with available observed MR configurations, while the shape and position of the constructed sound structure relative to a given MR pattern remain the same as those during the early stage of the same phenomenon near the wedge corner. We do not, of course, consider the incipient microscopic delay regime of the initiation of MR, as discussed by Walenta (1983).

There are two important reasons for analysing these uniform downstream sound

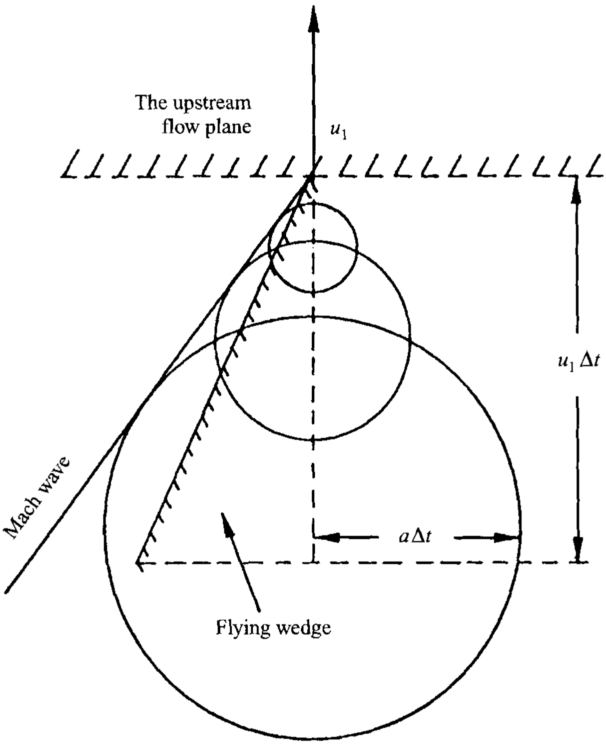

FIGURE 6. Sound generation by a flying wedge.

structures. First, by the nature of sound propagation, locally uniform sound waves inherently exist in both subsonic and supersonic flows when there is a source of sound generation initially present in the flow. For supersonic flows, their presence becomes more noticeable for a weak shock flow field than for a stronger one, where Rankine-Hugoniot jump relations are so apparently applicable to describing the latter. Secondly, the construction of the downstream sound structure is not meant to approximate a reflected shock wave in the first place; it is done to identify the physically acceptable branch of multi-valued shock reflection solutions of a pseudo-steady MR and to provide physical grounds for explaining possible reflected compression wave solutions, as previously discussed by Colella & Henderson (1990).

In the following, sound wave structures downstream of both weak and strong MR observed in Deschambault's (1984) interferometric experiments in air and those of a weak MR and a vNR in argon reported by Colella & Henderson (1990) are graphically constructed. Deschambault's work on pseudo-steady oblique shock wave reflections in air provides probably the best available interferometric pictures for the propagation of MR from the wedge tip for a wide range of M_s and θ_w. It is most instructive to construct the downstream sound waves induced by the triple point. This includes the first sound wave generated at the wedge corner. For the sake of clear illustration, only sound waves induced by the triple point are drawn. The constructed sound structures are shown in the same figures as the traced MR configurations obtained from the experiments so that direct comparisons between the two can be made. The sound construction is first explained in detail in §4.1 for three downstream cylindrically propagating sound disturbances induced by the leading triple point. These three propagating sound waves, originally centred at the selected locations depicted in §4.1, are used to draw the downstream sound waves for the other four cases. The propagation of sound waves generated at other locations along the slipstream can be similarly constructed.

4.1. *A weak Mach reflection with a forward-facing reflected shock solution for a 'small' wedge angle: $M_s = 1.37$, $\theta_w = 2°$, $\gamma = 1.4$*

Figure 7(*a*) shows a typical MR-like wave configuration for a weak incident shock reflecting from a 2° wedge traced from the finite-fringe interferogram of Deschambault (1984). The initial pressure is 749 Torr. The Mach stem is straight and perpendicular to the wedge surface, and χ is measured as 26.2°. The constructed sound wave structure is illustrated in the figure where three propagating sound disturbances, originally centred at locations given by the intersection of the incident shock with the triple-point path after the times $0, 0.5\Delta t$, and $0.75\Delta t$ when the incident shock first encounters the wedge, are shown. Δt is the time taken for the incident shock to travel from the wedge corner to its present location. The horizontal distances from the leading incident shock to these three original sound generation centres are shown in the figure, given by $u_1\Delta t$, $0.5u_1\Delta t$, and $0.25u_1\Delta t$, respectively. They are then pulled forward by the incident shock for distances given by $(u_1-u_2)\Delta t$, $0.5(u_1-u_2)\Delta t$, $0.25(u_1-u_2)\Delta t$, correspondingly. u_1, u_2 are the wave-fixed flow velocities ahead of and behind the incident shock wave, respectively. Thus, the trajectory of sound generation centres induced by the triple point lies in the same direction as the incoming flow velocity ahead of the reflected wave. The sound speed downstream of the incident shock is 1.11 times larger than that ahead of the shock.

It can be seen that the outermost sound wave originating from the wedge corner, when the incident shock first collides with it, completely matches with the observed reflected wave emanating from the triple point. The reflected wave angle is measured at 90.0°. The downstream sound structure, therefore, exhibits a limiting normal Mach wave configuration, and the observed reflected wave is, thus, verified to be a normal Mach wave supported by a sonically flying triple point. This sonic flow downstream of the incident shock relative to the triple point is checked by the oblique shock calculation, shown in figure 7(*b*), where the reflected shock polar degenerates to a point on the incident shock polar when the observed χ is used in the calculation. The self-similarity of this reflected normal Mach wave is evident from the two competing velocities downstream of the incident shock: the relative velocity of the downstream fluid and the propagating triple point, and the local fluid sound speed.

Actually, this reflected normal Mach wave represents a limiting configuration for the downstream, most forward-facing reflected shock or compression waves to be realized physically. The reason is that sound waves cannot coalesce into Mach waves that propagate upstream from the source of sound generation giving rise to them. On the other hand, the conventional three-shock theory requires a forward-facing reflected shock solution because the critical M_s is 4.01 for this 2° wedge in air. The calculated χ and ϕ_2 are 31.3° and 112.7°, respectively, so they differ significantly from the experimentally observed values. The shock polar solution is given in figure 7(*c*). It is interesting to note that if this predicted forward-facing reflected shock were to occur physically, the flow downstream of the Mach stem would have been supersonic at the Mach number 1.06! Therefore, forward-facing reflected shock or compression wave solutions of pseudo-steady MR should be ruled out physically on the basis provided by the above downstream sound consideration or simply by arguing that 90° is the maximum Mach angle possible in supersonic flows. In its place, for a 'small' reflecting wedge angle, a MR-like phenomenon with the reflected wave being a normal

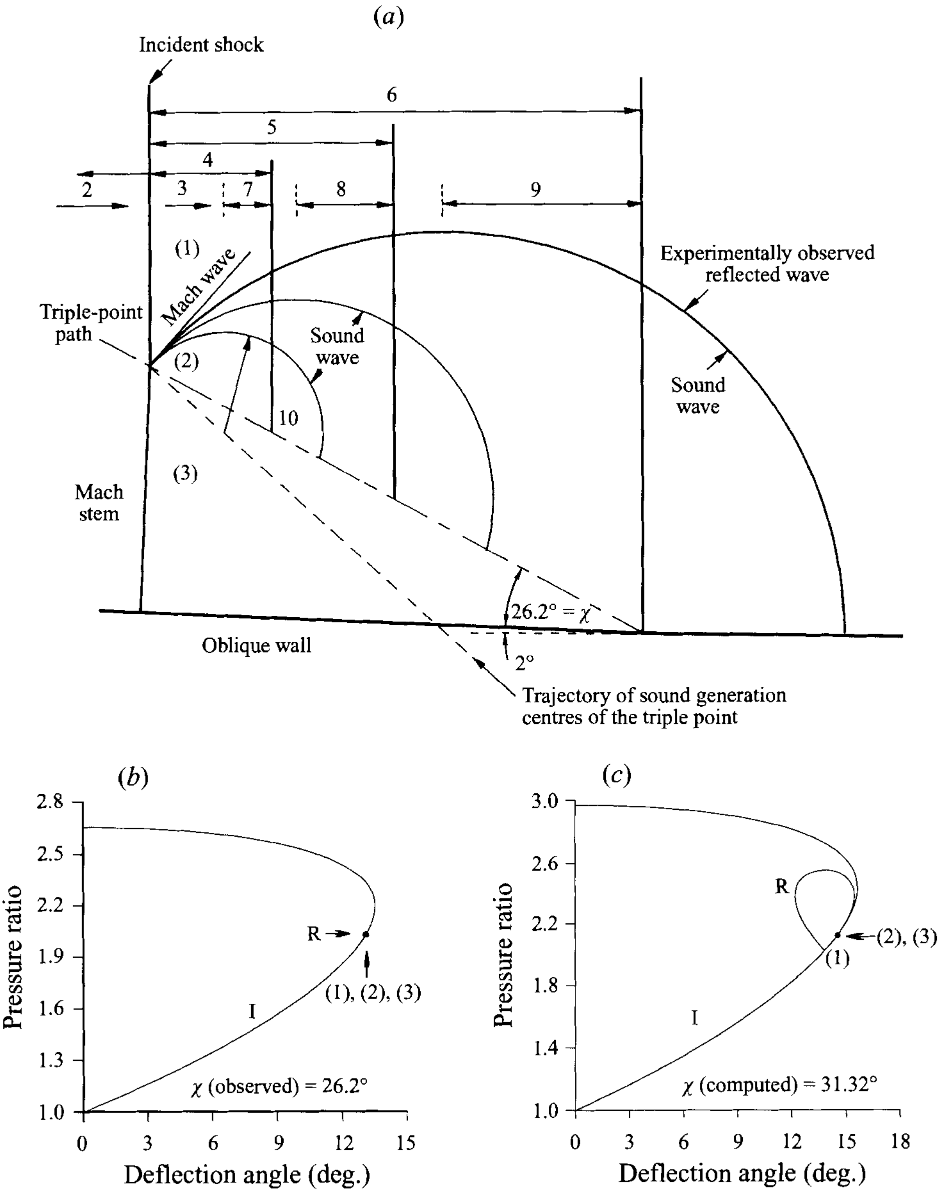

FIGURE 7. (*a*) Constructed sound wave structures downstream of observed pseudo-steady MR for $M_s = 1.37, \theta_w = 2°$ from Deschambault (1984). 1 and 2, $u_1 = 1.37a$; 3, $u_2 = 0.61u_1$; 4, $0.25u_1\Delta t$; 5, $0.5u_1\Delta t$ 6, $u_1\Delta t$; 7, $0.25(u_1 - u_2)\Delta t$; 8, $0.5(u_1 - u_2)\Delta t$; 9, $(u_1 - u_2)\Delta t$; 10, $0.25 \times 1.11a\Delta t$, where a is the upstream sound speed. (*b*) Degeneration of the reflected shock polar (R) to a single point on the incident shock polar (I) for this case. (*c*) The unphysical forward-facing reflected shock solution for this case.

Mach wave occurs. Moreover, §4.5 shows where a MR-like phenomenon occurs when the three-shock theory gives a physically acceptable backward-facing reflected shock solution. Therefore, the requirement of a forward-facing reflected shock solution of a pseudo-steady MR from the three-shock theory serves to provide a sufficient condition for the occurrence of MR-like phenomena. This single condition explains the two conditions for the occurrence of vNR proposed by Colella & Henderson (1990).

We shall analytically define the 'smallness' of a reflecting wedge angle of a pseudo-steady MR in §4.2, where a reflected backward-facing finite compression wave solution from a modified three-shock theory is introduced. It is shown there that a physically acceptable backward-facing reflected finite compression wave solution of a pseudo-steady MR is possible only when the wedge angle is larger than a limiting 'small' value for a given set of M_s and γ. The limiting small wedge angle for this case is 8.52°.

The finding of the reflected normal Mach wave provides the physical basis for using the acoustic criterion for describing the reflected wave in this MR-like phenomenon. It can be used for accurately predicting the χ of MR-like phenomena over 'small' wedge angles. This sonic flow criterion explains why Ben-Dor's (1978) technical approach for calculating χ agrees excellently with experiment only for low incident shock Mach numbers at small reflecting wedge angles. He assumed that the χ in weak MR over small wedge angles be large enough so that the flow downstream of the incident shock is at least sonic relative to the triple point. Application of this criterion for calculating χ gives 26.1° which is in almost exact agreement with the observed value, as expected. An exact formula is shown below, giving this acoustic-criterion-predicted χ as a function of γ, θ_w, and M_s, obtained directly from an equation given by AMES (1953, p. 9):

$$\chi^\circ = 90^\circ - \theta_w - \arcsin\left[\frac{(\gamma+1)M_s^4}{2\gamma M_s^4 + (3-\gamma)M_s^2 - 2}\right]^{1/2}.$$

We conclude discussion of this case by classifying a von Neumann reflection of the first kind (vNR- I) as a MR-like phenomenon over a 'small' reflecting wedge angle characterized by a reflected normal Mach wave. Properties of vNR over 'not-small' reflecting wedge angles are described in §§4.2 and 4.5.

4.2. *A weak Mach reflection with a forward-facing reflected shock solution for a 'not-small' wedge angle:* $M_s = 1.48, \theta_w = 14.58^\circ, \gamma = 5/3$

Figure 8(*a*) depicts a MR-like wave configuration for a weak incident shock reflecting from a 14.58° wedge traced from the schlieren photograph reported by Colella & Henderson (1990). The initial pressure here and that in §4.3 were the ambient pressure. The observed χ and ϕ_2 are 15.6° and 90°, respectively. The estimated accuracy for angle measurement from the photographs of Colella & Henderson is within $\pm 0.5^\circ$ for the reflected wave and ± 0.15 for all other wave angles. The contact discontinuity, which appears slightly broad, can be seen from the schlieren picture. The slipstream line shown in the figure is a mean line drawn through the centre of the observed discontinuity. The constructed trajectory of sound generation centres of the triple point falls 1.7° below the slipstream, so the observed reflected wave is backward-facing. One can regard this 1.7° deflection as the angle of an effective flying wedge induced from the propagating triple point. Fluid particles upstream and downstream of the reflected wave move along the lower and upper surfaces of this effective flying wedge, respectively. Thus, it can be viewed as the source of sound generation, from the upper surface of which reflected disturbances are constantly generated as the triple point propagates upstream. However, as will be illustrated, the coalescing process of sound-induced Mach waves in this case is not complete, and a reflected finite compression wave is formed instead.

It can be seen from figure 8(*a*) that the outermost sound wave, originating from the wedge corner, falls within the immediate vicinity of the observed reflected wave.

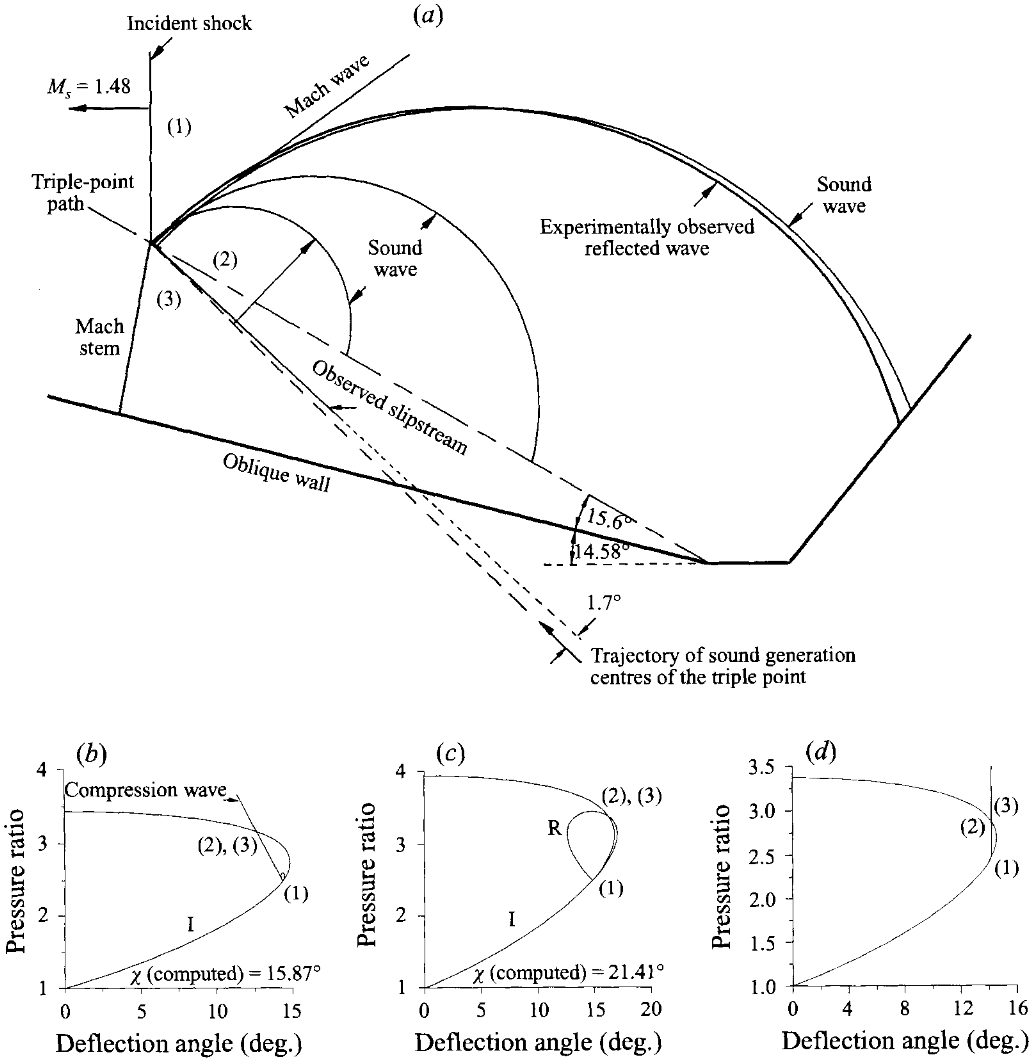

FIGURE 8. (a) As figure 7(*a*) but for $M_s = 1.48$, $\theta_w = 14.58°$ from Colella & Henderson (1990). (*b*) The (P, θ)-polar solution for this case. (*c*) The unphysical forward-facing reflected shock solution for this case. (*d*) The shock polar solution of the limiting small reflecting wedge angle for this case, with $\theta_{w,small} = 7.96°$.

Although the sound structure near the triple point still somewhat resembles a limiting normal Mach wave configuration, sound waves generated by the triple point do not form a normal Mach line there. Actually, these sound waves constitute a leading Mach wave stemming from the triple point with a 79.4° Mach angle. This Mach angle corresponds to a very weak supersonic flow downstream of the incident shock with the Mach number being 1.017. An oblique shock calculation using the observed χ gives the same flow Mach number. The close agreement shows the accuracy of our constructed sound structure. Now, this very weak supersonic flow downstream of the incident shock may not allow coalescence of Mach waves into a reflected shock wave to occur. This is supported by the fact that the constructed outermost sound wave matches closely with the observed reflected wave and that the induced Mach wave lies close to the shape of the observed reflected wave in a large region from the triple point. Furthermore, it can be seen from Colella & Henderson's schlieren photograph that the observed reflected wave appears to have a finite thickness. This type of observed reflected wave was discussed by Sakurai *et al.* (1989) and it was suggested by them that the thickness of it is by no means negligible. The experimentally observed reflected wave shown in the figure is a mean line drawn through the centre of the edges of the observed reflected wave pattern. Most significantly, it is found that the maximum flow deflection achievable through an oblique shock with this flow is only 0.10°, which is more than an order of magnitude smaller than the required 1.7° deflection across the observed reflected wave. Therefore, Rankine–Hugoniot jump conditions are not applicable for this reflected wave near the triple point. It is interesting to note here the practically zero entropy increase across a normal shock with this flow Mach number of 1.017. These observations provide convincing evidence that a reflected backward-facing finite compression wave should occur to turn the downstream flow parallel to the slipstream. This description of the observed reflected wave is in line with Colella & Henderson's (1990) numerical finding of a band of reflected smoothly distributed compression waves of finite thickness in a MR-like phenomenon. The weak oblique shock approximation linearly relating finite changes between the pressure and flow deflection across a weak oblique shock, as discussed by Liepmann & Roshko (1957) valid for small deflection, can be used to describe this reflected compression wave of small finite thickness:

$$\frac{\Delta P}{P} = \frac{\gamma M_1^2}{(M_1^2-1)^{1/2}}\Delta\theta$$

where M_1 is the incident flow Mach number of the reflected wave.

The thickness of this reflected compression wave is characterized by a small but finite $\Delta\theta$, so that the weak oblique shock approximation is applicable. A key step in deriving the above approximation is the substitution of the shock angle by Mach angle in this linearized pressure–deflection oblique shock relation. Therefore, the reflected finite compression wave can be considered as composed of a series of weak oblique shock waves of the same strength. Each weak oblique shock wave may then be viewed as made up of many isentropic converging Mach waves. The incorporation of this (P,θ)-relation for a reflected finite compression wave, in place of jump relations of a reflected shock wave, into otherwise the same three-shock theory is expected to provide an adequate description for this observed MR-like phenomenon.

Ben-Dor (1992) suggested this simple gasdynamic consideration for the reflected compression wave of a vNR. However, the (P,θ)-polar solution discussed by him faces forward of the triple point. The polar solution should be obtained by drawing a line with the gradient above calculated, in the direction of decreasing deflection and increasing pressure, from the incident shock polar. The incident flow Mach number M_1 ahead of the reflected wave is determined by iteratively specifying χ, thereby M_1, until the normal, straight Mach stem boundary condition is satisfied. A schematic drawing of the (P,θ)-polar solution of this MR-like phenomenon is shown in figure 8(*b*), where the intersection (points 2 and 3) of the linear compression polar and the incident shock polar gives the flow properties sought behind the backward-facing reflected compression wave and the Mach stem. The corresponding reflected shock polar is also shown in the figure. It can be seen that the tiny reflected shock polar does not intersect the incident polar except for the

Mach line degeneracy point. The computed χ, 15.9°, and the angle between the slipstream and trajectory of the triple-point sound generation centres, 1.7°, from this modified three-shock theory, are in close agreement with the experimentally observed values. The above analyses therefore experimentally and analytically demonstrate the existence of a reflected backward-facing finite compression wave in this MR-like phenomenon over a 14.58° wedge. Since the computed flow Mach number downstream of the incident shock is near sonic, it is worthwhile to calculate χ using the acoustic criterion of vNR-I, identified in §4.1, for this MR-like phenomenon. One obtains $\chi = 15.0°$, which is only 0.6° smaller than the experimental value.

The conventional three-shock theory, on the other hand, requires a forward-facing reflected shock solution as given in the shock polar in figure 8(c). The predicted χ and ϕ_2 are 21.4° and 97.2°, respectively, so they clearly do not agree with the observed values. Thus, it is confirmed here that theoretically predicted forward-facing reflected shock solutions of pseudo-steady MR should be ruled out physically. In their place, for a 'not-small' reflecting wedge, a MR-like phenomenon with the reflected wave being a backward-facing finite compression wave occurs, and it is classified as the second kind von Neumann reflection (vNR-II).

It is worth noting that the effect of viscous displacement thickness on the flow downstream of the incident shock by the shear layer (rather than a slipstream) may be argued not small in comparison with the 1.7° turning angle. This consideration, however, will not change the main features of the analysis because the downstream flow is verified to be very weak supersonically, having $M_1 = 1.017$. That is, only a backward-facing reflected finite compression wave solution can provide a predicted χ in reasonably close agreement with the experimentally observed value.

The necessity of classifying a reflecting wedge angle of a pseudo-steady MR as 'small' arises from the occurrence of the physically unrealistic forward-facing reflected finite compression wave solution in the above-discussed modified three-shock theory when it becomes sufficiently small for a given combination of M_s and γ. A 'small' reflecting wedge angle of a pseudo-steady MR is therefore defined as, for a given set of M_s and γ, a wedge angle below which a backward-facing reflected finite compression wave solution satisfying the normal, straight Mach stem boundary condition is theoretically unobtainable. Figure 8(d) illustrates the (P, θ)-polar solution for the limiting small wedge angle of vNR-II for the present case, where the flow downstream of the incident shock is across a reflected normal compression wave. The computed 'small' angle is 7.96°.

It is important to establish the regimes of vNR-I and vNR-II. Curve (a) in figure 9 shows the calculated limiting small reflecting wedge angles for $\gamma = 7/5$ and $1.0 < M_s \leqslant 4.0$. VNR-I occurs when the wedge angle falls below this curve. Curve (b) is the computed boundary separating regimes of backward- from forward-facing reflected shock solutions of pseudo-steady MR. This boundary terminates at $M_s = 1.11$, where it meets the transition boundary dividing regimes of RR from MR and vNR. Classical Mach reflections with reflected backward-facing shock waves are possible only when the reflecting wedge angle is above curve (b), while vNR-II take place in the region between curves (b) and (a). Figure 10 shows the comparison of calculated limiting small reflecting wedge angles between $\gamma = 7/5$ and $5/3$ for $1.0 < M_s \leqslant 4.0$. It can be seen that the two theoretical 'small' reflecting wedge angles are practically indistinguishable in the range $1.0 < M_s \leqslant 1.7$. The 'small' wedge angles for $\gamma = 5/3$ then become slightly larger than those for $\gamma = 7/5$ when $M_s \geqslant 1.7$.

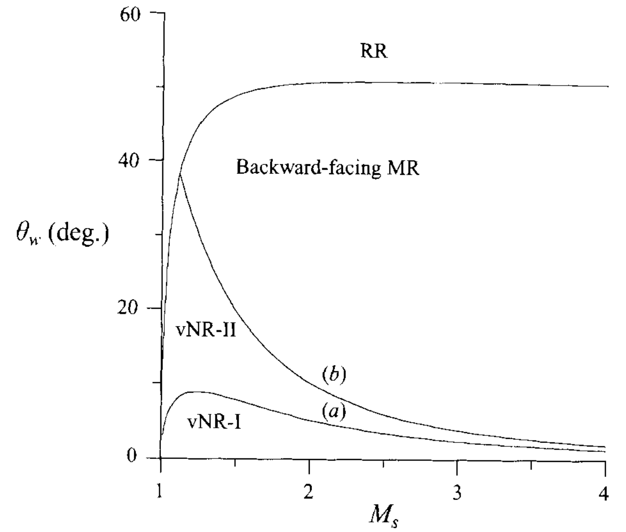

FIGURE 9. Boundaries separating regimes of vNR-I, vNR-II, backward-facing MR, and RR for $\gamma = 7/5$.

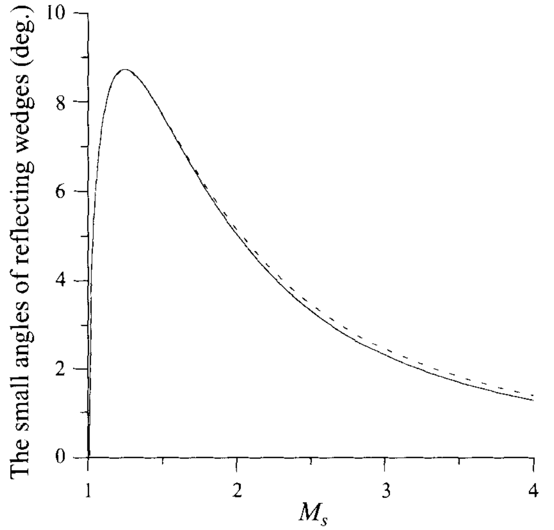

FIGURE 10. Theoretical limiting small reflecting wedge angles of pseudo-steady MR for $\gamma = 7/5$ (solid line) and 5/3 (dashed line).

4.3. *A weak Mach reflection with a backward-facing reflected shock solution:* $M_s = 1.47, \theta_w = 34.6°, \gamma = 5/3$

A typical weak MR wave configuration, where the three-shock theory gives a backward-facing reflected shock solution, is shown in figure 11. This wave configuration is traced from the schlieren picture of Colella & Henderson (1990). The Mach stem is straight and perpendicular to the wedge surface, and the observed χ is 4.5°. The observed slipstream, although having finite thickness, can be seen to be quite straight near the triple point. It is shown by the solid line drawn equidistant between the straight portion of this observed discontinuity. The constructed sound

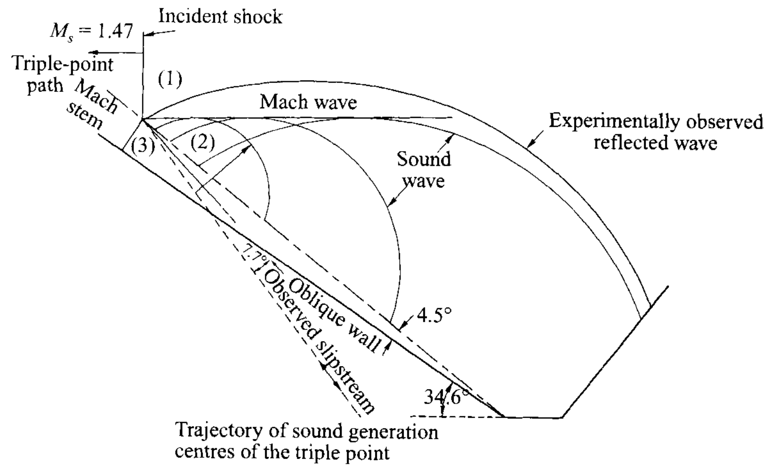

FIGURE 11. Constructed sound wave structures downstream of observed pseudo-steady MR for $M_s = 1.47$, $\theta_w = 34.6°$ from Colella & Henderson (1990).

structure downstream of the incident shock shows that the Mach wave induced by the propagating triple point is formed significantly inside the observed reflected wave. The trajectory of sound generation centres of the triple point falls 7.7° below the slipstream. These observations suggest that a reflected shock is required to turn the downstream flow parallel to the slipstream.

However, the conventional three-shock theory predicts χ and the angle between the trajectory of the triple-point sound generation centres and the slipstream to be 8.0° and 4.7°, respectively. Thus the theory fails to describe this weak MR, even though it does provide a backward-facing reflected shock solution. If one uses, instead, the measured χ to calculate angles between various discontinuities in the vicinity of the triple point, one obtains the angles between the incident and reflected shocks (ω_{ir}) and between the reflected shock and slipstream (ω_{rs}) to be 55.8° and 74.0°, respectively. Therefore, the calculated ω_{ir} is 7.7° smaller than the observed angle, while the computed ω_{rs} becomes 1.5° larger than the measured value. This result, however, is opposite to what Ben-Dor (1987) concluded for the effect of viscous displacement thickness in the prediction of various wave angles of a pseudo-steady MR. For an incident shock of Mach number 2.71 reflecting off a 47.1° wedge, he used the experimentally observed χ in the three-shock theory and found that the calculated ω_{ir} is 5.0° larger than the experimental value while the computed ω_{rs} is 4.7° smaller than the observed value. Ben-Dor then obtained excellent agreement between the experiment and theory when the viscous displacement effect along the slipstream is integrated into the three-shock theory. Therefore, the failure of the three-shock theory to describe this weak MR cannot be attributed to the viscous effect according to the result from Ben-Dor's (1987) analysis.

In fact, this large discrepancy between the predicted and measured χ can be expected to exist for weak MR with theoretically predicted backward-facing reflected shock solutions, based on the computed and measured χ presented in figure 3. We have shown in §4.2 that the large discrepancy there can be very well explained by replacing the physically unacceptable forward-facing reflected shock solution with a backward-facing reflected finite compression wave solution in otherwise the same three-shock theory. Now the only difference in the experimental conditions between this and §4.2 is the wedge angle being considerably larger here (the difference in M_s

is too small). It is shown in figure 9 that, when M_s is greater than 1.11, a MR will transform into a vNR-II as the wedge angle decreases. Therefore, a key question needs to be addressed before adequate explanations can be provided for the existing large discrepancies between the three-shock theory and experiment for weak MR, when the theory gives backward-facing reflected shock solutions: Is the problem related to the process of a continuously smooth transition from MR to vNR-II, as is evident from the comparison between the measured and computed χ given in figure 3? This suggestion is also supported by the result in §4.5 where a strong incident shock reflecting from a 'not-small' wedge angle, when the three-shock theory provides a backward-facing reflected shock solution, is a vNR-II.

4.4. *A strong Mach reflection with a forward-facing reflected shock solution for a 'not-small' wedge angle: $M_s = 3.93, \theta_w = 2°, \gamma = 7/5$*

Figure 12(*a*) depicts a MR-like wave configuration for a strong incident shock reflecting from a 'not-small' 2° wedge traced from Deschambault's infinite-fringe interferometric picture. The initial pressure is 15 Torr. The Mach stem is straight and perpendicular to the wedge surface, and a thin slipstream is visible from the picture. The observed χ and ϕ_2 are 22.6° and 90.0°, respectively. The theoretical limiting small reflecting wedge angle is only 1.46° for this case. The classical three-shock theory gives a forward-facing reflected shock solution for this MR-like phenomenon. The computed χ and ϕ_2 are 22.78° and 90.3°, respectively. It is noteworthy that the calculated χ agrees closely with the experiment when the computed reflected shock is forward-facing. Actually, a vNR should develop according to the sufficient condition for its existence. This is supported by the constructed sound structure where the outermost sound wave exactly matches with the observed reflected wave in regions not far from the triple point. It is also consistent with the identical match between the observed slipstream and the trajectory of sound generation centres of the triple point. The calculated χ, 22.43°, using the downstream acoustic criterion, is in close agreement with the observed value. On the other hand, the fact this wedge angle is not 'small' implies that a reflected backward-facing finite compression wave solution is obtainable. This is shown in a locally enlarged shock polar solution of vNR-II, given in figure 12(*b*), where the origin of the reflected linear (P, θ)-compression polar lies very close to the sonic point of the incident shock polar. The corresponding very tiny reflected shock polar is also shown to highlight the weakness of this downstream supersonic flow ($M_1 = 1.0005$). The computed χ is 22.44°, which is practically undistinguishable from the value predicted by the downstream acoustic criterion. This case serves to illustrate the inherent nonlinearity of MR phenomena where the difference in χ obtained from the experiment and the three different theoretical reflection solutions – forward-facing shock wave, backward-facing compression wave, and normal Mach wave – are within 0.4°.

4.5. *A strong Mach reflection with a near-forward-facing reflected shock solution for a 'not-small' wedge angle: $M_s = 2.03, \theta_w = 10°, \gamma = 7/5$*

Figure 13(*a*) shows a MR-like phenomenon for a strong shock reflecting from a 'not-small' 10° wedge traced from the finite-fringe interferogram of Deschambault. The limiting small reflecting wedge angle is 5.0° for this case. The initial pressure is 50 Torr. The Mach stem is straight and perpendicular to the wedge surface. The thin slipstream can be identified from the interferometric picture. The observed χ and ϕ_2 are 18.5° and 90.0°, respectively. The conventional three-shock theory gives a near-forward-facing reflected shock solution, as shown in figure 13(*b*). The computed

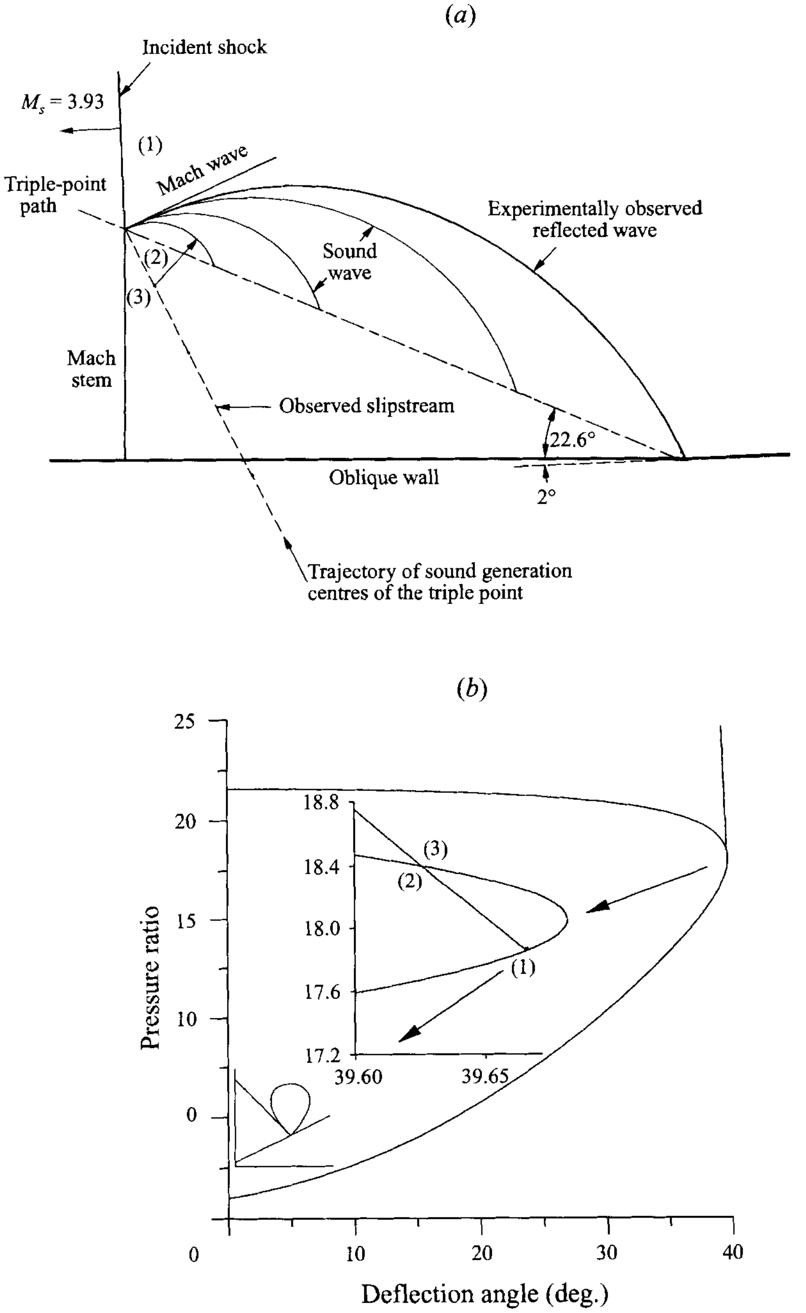

FIGURE 12. (*a*) Constructed sound wave structures downstream of observed pseudo-steady MR for $M_s = 3.93$, $\theta_w = 2°$ from Deschambault (1984). (*b*) The reflected backward-facing finite compression wave solution for this case.

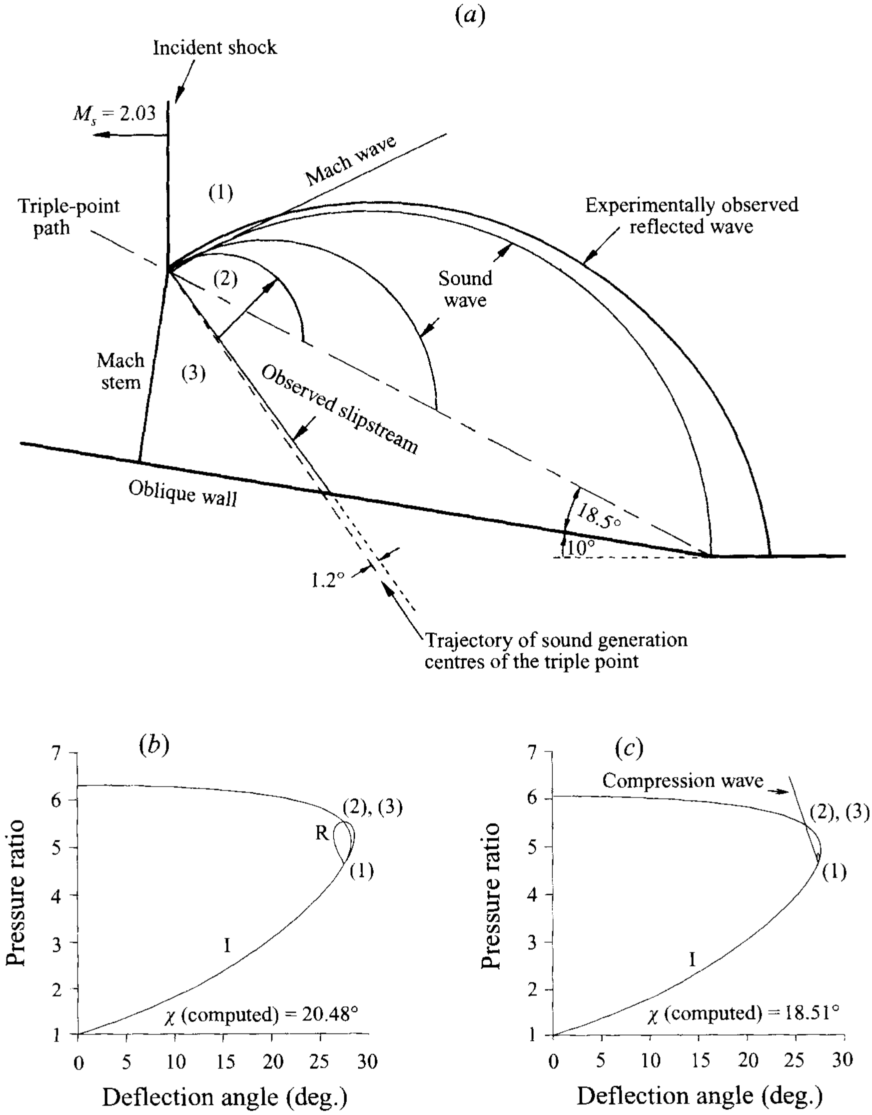

FIGURE 13. (*a*) As figure 12(*a*) but for $M_s = 2.03$, $\theta_w = 10°$. (*b*) The near forward-facing reflected shock solution for this case. (*c*) The (P, θ)-polar solution for this case.

χ and ϕ_2 are 20.5° and 89.6°, respectively. Thus, it is shown that a large discrepancy between the three-shock theory and experiment also exists for a strong MR over a 'not-small' wedge angle, when the theory provides a backward-facing reflected shock solution.

The constructed sound structure shows that the outermost sound wave falls just slightly inside the observed reflected wave near the triple point. Deschambault's interferogram reveals that there is a small region immediately behind the front of the reflected wave where the slope of the fringes changes rapidly. Therefore, this observed reflected wave has finite thickness, and the observed reflected wave is a mean line drawn through the centre of the reflected interferometric wave pattern. The flow Mach number downstream of the incident shock computed from the observed χ is 1.018. In addition, it can be seen from the figure that the flow deflection across the observed reflected wave is 1.2°. Thus, the characteristics of the flow field downstream of the incident shock, responsible for the formation of the reflected wave, are essentially the same as those discussed in §4.2. This allows the application of the modified three-shock theory, also discussed there, for this MR-like phenomenon. The calculated χ and angle between the slipstream and the direction of the incident flow before the reflected wave are 18.5° and 1.3°, respectively, so they are in excellent agreement with the experimental values. The computed (P, θ)-polar solution is given in figure 13(c). We have thus shown that this observed strong MR belongs to vNR-II. However, unlike the vNR-II in §§4.2 and 4.4, its occurrence here is not predicted by the sufficient condition for its existence. This finding is of significance in that it shows Nature's choice of a weaker possible vNR-II solution, when the three-shock theory provides a physically acceptable near-forward-facing reflected shock solution. Consequently, one can infer that the large discrepancies between the classical three-shock theory and experiment for strong MR, when the theory gives near-forward-facing reflected shock solutions, can be attributed to the occurrence of vNR-II or the continuously smooth transition between MR and vNR-II.

Finally, it is instructive to calculate the χ of this observed vNR-II using the downstream acoustic criterion of vNR-I. The computed result, $\chi = 17.86°$, deviates from the experiment by only 0.6°. This illustrates the applicability of the acoustic approximation for the reflected waves of vNR-II, even when the three-shock theory gives a physically acceptable reflection solution. Since a computed χ from the downstream acoustic criterion decreases linearly as θ_w increases for a given set of M_s and γ, one expects the prediction of χ for vNR-II over not large θ_w can be quite accurately determined using this approximation. Two series of calculations for χ, using both the exact (P, θ)-polar solution of vNR-II and the downstream acoustic criterion, were performed for $\theta_w = 10°$ and 15° with $\gamma = 7/5$ and $1.0 < M_s \leqslant 3.0$. The difference in the two computed χ is plotted in figure 14. It is found, within the regime determined by forward-facing reflected shock solutions of MR for θ_w up to 15° and M_s not smaller than 1.09, that the predicted χ of vNR-II using the acoustic approximation gives results that differ from those obtained from the exact (P, θ)-polar solution by less than 1.49°. The boundary separating vNR-II from backward-facing MR is also shown in the figure.

5. Conclusions

It is shown that the sound structures downstream of the incident shock waves of pseudo-steady MR provide physical bases for using the acoustic criterion or the weak oblique shock approximation for describing reflected waves in MR-like phenomena. Several important findings that successfully explain observed MR-like phenomena and identify key features of vNR are obtained.

For experimentally observed MR-like phenomena over 'small' wedge angles, where the classical three-shock theory requires forward-facing reflected shock solutions, downstream sound wave structures show that the observed reflected wave is a limiting normal Mach wave. The acoustic criterion for the reflected wave is thus identified, and it gives almost exact predictions for the observed triple-point trajectory angles, χ. The 'smallness' of a reflecting wedge angle of a pseudo-steady MR is defined as, for a given set of M_s and γ, the wedge angle below which a reflected backward-facing finite compression wave solution of a pseudo-steady MR satisfying the normal, straight Mach stem boundary condition is theoretically unobtainable.

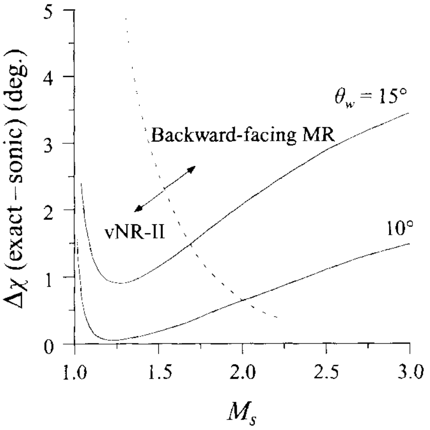

FIGURE 14. Difference in the prediction of χ between using the downstream acoustic criterion and from the exact polar solution of vNR-II for $\theta_w = 10°$ and 15°.

For experimentally observed MR-like phenomena over 'not-small' wedge angles, where the three-shock theory requires forward-facing reflected shock solutions, downstream sound structures show that the observed reflected waves exhibit characteristics of backward-facing compression waves of small finite thickness. The application of the weak oblique shock approximation to the reflected wave in otherwise the same three-shock theory for the observed MR-like phenomena gives the predicted χ in close agreement with the experiments.

It is concluded that forward-facing reflected shock solutions of pseudo-steady MR should be ruled out physically because sound waves cannot coalesce into Mach waves that propagate upstream of the triple point. In their place, MR-like phenomena occur with the reflected waves being normal Mach waves or finite compression waves for 'small' or 'not-small' reflecting wedge angles, respectively, and they are classified as vNR-I or vNR-II, respectively. Boundaries separating vNR-I, vNR-II, and backward-facing MR regimes are determined.

A comparison between theory and experiment, when the three-shock theory provides physically acceptable near-forward-facing reflected shock solutions, reveals Nature's choice of a weaker possible reflection solution, i.e. a reflected backward-facing finite compression wave, instead of a reflected backward-facing shock wave. This finding suggests that the large discrepancies between the classical theory and experiment, when the theory gives near-forward-facing reflected shock solutions, can be attributed to the occurrence of vNR-II or the continuously smooth transition between MR and vNR-II. A sufficient condition for the existence of vNR is then given by the requirement of forward-facing reflected shock solutions of pseudo-steady MR from the classical three-shock theory.

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